

# The suppression of wind-generated waves by a surface film

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(Received 8 July 1967)

In the presence of a surface-active agent waves generated on the surface of a liquid by a wind may be stabilized. This problem is explored, and in particular the critical wind velocity is calculated for wave inception in the presence of such a film.

The dominant parameter associated with the surface layer is the surface elasticity  $\chi$ ; the 'inextensible film' of Lamb is obtained as a limit as  $\chi \rightarrow \infty$ . Such a value is hypothetical as for real films  $\chi$  is not large, being in the range 0 to 80 dyne/cm approximately. Nevertheless, we show that damping exceeding that of the inextensible film can be obtained for short wavelengths, and that the critical wind speed for capillary ripples can be increased by a factor of ten in the presence of a film of experimentally attainable characteristics.

The reason for the effectiveness of films of low  $\chi$  is that the damping in the liquid is related to  $\chi$  and the wavelength  $\lambda$  in such a way that for small  $\lambda$ , damping is a maximum for small  $\chi$ .

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## 1. Introduction

The calming effect of small quantities of oil on wind-driven ocean waves has been known since classical times. Plutarch (A.D. 95) and the Venerable Bede (A.D. 731) mention its use, and Pliny (A.D. 77) describes how pearl fishers used oil to damp small ripples to obtain a clearer underwater light. Benjamin Franklin (1774) gives a number of interesting applications to do with fishing and seafaring in general. He also describes a number of his own experiments carried out on various lakes and ponds in England, and off the sea coast near Portsmouth. The inland waters he was able to make 'as smooth as a looking glass'; on a pond in Clapham Common a teaspoonful of oil had this effect on an area of half an acre. At sea the ground swell continued but the small capillary waves disappeared.

Experiments on the sea were also described by Aitken (1882) who in addition carried out a number of simple laboratory experiments. He postulated that the prime effect of the oil was to suppress the formation of infinitesimal ripples, thereby 'strangling the wave in its birth'. In a qualitative way he attributed the suppression to variations in surface tension caused by the distortion of the surface by the wave. This view was also proposed by Reynolds (1880) (see Lamb 1932, p. 631).

More recently, van Dorn (1953) found that waves in an 800 ft. model-yacht

basin could be suppressed by a soluble detergent. He records that even at high wind speeds wave damping occurred, although above 15 m/sec, the surface film tended to be blown away.

Wind tunnel experiments have been carried out by Keulegan (1951) and Fitzgerald (1963). Both of these authors used surface active agents for suppressing wave formation, to enable them to measure velocity profiles over smooth water surfaces. Keulegan more or less stumbled across the effect when traces of soap were left in the apparatus; with a perfectly clean system waves formed above 3 m/sec, but with the soap, even at 12 m/sec the surface was free of waves. Fitzgerald, who was primarily concerned with reduction of evaporation from water storages, used substances such as cetyl alcohol which form a coherent film on the gas-liquid interface having substantial resistance to the passage of water molecules.

The purpose of this paper is to explain how a thin surface film of only molecular thickness can bring about the dramatic calming effect described by Franklin and others, and to find the wind velocity needed to raise waves on a contaminated surface (i.e. to find the neutral stability diagram for the system). The results are also relevant to other fields such as chemical engineering, where problems involving the gas-liquid interface occur (e.g. in two-phase flow in pipes, wetted wall columns, etc.).

In forming a convenient mathematical description, the oil (or other suitable substance) will be regarded as forming an adsorbed monomolecular layer on the surface of the water. Such films have certain well-defined elastic-viscous properties (as well as a host of conjectural ones, see Goodrich (1962)), but in this work only one will be considered, the so-called Gibbs surface elasticity  $\chi$ . In §2, infinitesimal waves on the surface of the liquid are treated by the linearized theory of Lamb (1932), following Miles (1962), but with appropriate allowances made for Gibbs elasticity modulus in the boundary conditions (Dorrestein 1951; Levich 1962). The eigenvalue relation for the complex wave velocity  $c_i$  which includes the gas phase stress contributions acting on the surface, is obtained.

Miles (1967*a*) considered briefly the calculation of the minimum wind velocity necessary to cause waves on a surface contaminated with an 'inextensible film' ( $\chi \rightarrow \infty$ ). His order-of-magnitude calculations showed that for long waves (6 m) the critical speed of the wind was increased by a factor of ten, while for capillary waves the factor was about two. This does not agree well with observation which indicates that a surface film only affects capillary ripples. In fact we show that for short waves very small values of  $\chi$  (such as are available experimentally) produce much higher damping than with  $\chi = \infty$  and also that no known surface film could affect waves of length greater than about 1 m.

The surface gas-phase stresses are calculated in §3 by the method of Benjamin (1959) and Miles (1962), and an improved method is described for the case where the critical layer occurs in the transition region just outside the laminar sublayer, where Miles's method is somewhat inaccurate. The eigenvalue problem is solved, thereby permitting the stability characteristics of the wave to be evaluated. The minimum velocity for wave inception is found to be increased by a

factor of up to ten for short waves and with surface films of realistic characteristics. This result is discussed in §4.

## 2. The liquid

We take an infinitely deep liquid of density  $\rho$ , kinematic viscosity  $\nu$  and surface tension  $\sigma$  that is bounded above by the surface wave

$$y_0 = y_0(x, t) = a e^{ik(x-ct)} \tag{2.1}$$

in the Cartesian co-ordinates  $x, y$ . The amplitude  $a$  is such that  $ka \ll 1$ ,  $k$  being the wave number. The wave velocity is complex ( $= c_r + ic_i$ ) and it is assumed that the wave Reynolds number is large so that  $c_i \ll c_r$ , and the linearized Navier–Stokes equations may be used.

If  $p_s, \tau_s$  are the normal and tangential stresses exerted by the gas at the surface, then these must be equal to the corresponding stresses  $p_{yy}, \tau_{xy}$  in the liquid at  $y_0$ . The gas stresses are of the form

$$p_s = \rho_a P_s y_0, \quad \tau_s = \rho_a T_s y_0, \tag{2.2}$$

where  $P_s, T_s$  are in general complex;  $\rho_a$  is the density of the gas. The stresses  $p_{yy}, \tau_{xy}$  are functions not only of the liquid bulk properties, but also of the visco-elastic properties of the surface film. A large number of these has been proposed (Goodrich 1962), but only two have received substantial experimental investigation. These are the Gibbs surface elasticity or compressibility modulus, and surface viscosity (Davies & Rideal 1961). Now Dorrestein (1951) and Davies & Vose (1965) have shown that the surface viscosity has in practice a negligible effect on damping of ripples. Accordingly we will ignore all visco-elastic properties of the film except the surface elasticity. For the present the film is assumed to be insoluble; the effect of solubility is discussed below.

By treating the surface elasticity in terms of surface tension, the stress relations in the liquid at the interface may be written

$$p_{yy} = -p + 2\mu \left( \frac{\partial v}{\partial y} \right)_{y_0} + \sigma \left( \frac{\partial^2 y_0}{\partial x^2} \right) - \rho g y_0, \tag{2.3}$$

$$\tau_{xy} = \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)_{y_0} - \frac{\partial \sigma}{\partial x}, \tag{2.4}$$

with  $p$  being obtained by integration of the  $x$ -momentum equation.

Following Levich (1962) we evaluate  $\partial\sigma/\partial x$  by writing it in the form  $(\partial\sigma/\partial\Gamma)(\partial\Gamma/\partial x)$ , where  $\Gamma$  is the concentration of the surfactant. A mass balance over the surfactant yields

$$\frac{\partial\Gamma}{\partial t} + \frac{\partial(u_0\Gamma)}{\partial x} = D \frac{\partial^2\Gamma}{\partial x^2}, \tag{2.5}$$

where  $\Gamma = \Gamma_0 + \Gamma'$  ( $\Gamma' \ll \Gamma_0$  to allow linearization),  $\Gamma_0$  being the concentration in the undisturbed surface. The right-hand side represents diffusion of surfactant through the substrate and is usually small since  $D$  is of order  $10^{-5}$  cm<sup>2</sup>/sec, and hence the diffusion time  $k^2/D$  is generally much greater than the period of oscillation, of order  $k/c$ . The solution of (2.5) yields  $(\partial\sigma/\partial x) = -i\rho\mathcal{D}cu_0$ , where a dimensionless group

$$\mathcal{D} = k\chi/\rho c^2 \tag{2.6}$$

has been defined, and the surface compressibility  $\chi = -\Gamma_0(\partial\sigma/\partial\Gamma)$ .

By following Miles's (1962) method with the altered boundary conditions, the resulting eigenvalue relation is

$$c_i = \frac{c_0}{2} \mathcal{D} \left[ -\frac{2i}{R_w} - \frac{2iK}{R_w} + \mathcal{D}K + \frac{\rho \hat{a}}{\rho c^2 k} (P_s + iT_s K) \right], \quad (2.7)$$

where  $K = [1 - \mathcal{D}(R_w/2)^{\frac{1}{2}}(1-i)]^{-1}$  and  $c_0 = (g/k + \sigma k/\rho)^{\frac{1}{2}}$  is the velocity of a surface wave on an inviscid liquid. Equation (2.7) reduces to the pure liquid case when  $\mathcal{D} = 0$ ; for  $\mathcal{D} \rightarrow \infty$ ,  $K_r \rightarrow K_i \rightarrow -[\mathcal{D}(R_w/2)^{\frac{1}{2}}]^{-1}$  so that providing  $R_w$  is large the 'inextensible film' result of Lamb (1932) is obtained. But in practice  $\mathcal{D}$  is never large;  $\chi$  has typical values up to about 80 dynes/sec and since surface damping is most effective for short waves,  $c_0^2 = \sigma k/\rho$ ; hence  $\mathcal{D} \simeq \chi/\sigma$  which is of order unity.

A curious feature of (2.7), together with the definition of  $K$ , is that the damping in the liquid is increased whether  $\chi$  is negative or positive. For insoluble surface films,  $\chi$  is generally positive. In this case (disregarding the gas phase for the moment)  $|c_i|$  has a maximum at  $\mathcal{D}(R_w/2)^{\frac{1}{2}} = 1$ , when  $K_r = 0$  and  $K_i = -1$ . The value of the maximum is for large  $R_w$ ,  $c_0/(2R_w)^{\frac{1}{2}}$  which is twice the value for the inextensible film; thus the rate of viscous dissipation is also twice the inextensible film value. The existence of this peak was first pointed out by Dorrestein (1951) and has since been verified experimentally by Davies & Vose (1965).

If  $\chi$  is negative on the other hand, the viscous damping increases monotonically to the inextensible film result  $\mathcal{D} = -\infty$ . There are a few soluble substances for which  $\chi < 0$  (it is thermodynamically impossible for  $\chi < 0$  for insoluble films). The most common are sucrose and salts such as magnesium sulphate, and sodium, potassium and calcium chlorides. The above analysis is applicable to soluble compounds (see below) so we conclude that any substance for which surface tension increases or decreases with concentration will increase the viscous damping at the surface of the liquid. This suggests, incidentally, that short waves on the surface of the ocean may be slightly damped by the presence of the salt.

In the case of a soluble film (the undisturbed bulk concentration being  $C_0$ ) a new values of  $\chi$  is found, say  $\chi'$  which is related with the former by the relation (Lucassen & Hansen 1966)

$$\chi' = \chi \left[ 1 + (1+i) \left( \frac{D}{2kc} \right)^{\frac{1}{2}} \left( \frac{\partial C_0}{\partial \Gamma_0} \right) \right]^{-1}. \quad (2.8)$$

In most practical cases however,  $\chi' \simeq \chi$ .

### 3. The surface stresses

The stresses  $P_s, T_s$  exerted by the gas may be found by the method of Benjamin (1959) as presented by Miles (1962):

$$P_s = U_0' \phi_0 + c \phi_0', \quad (3.1)$$

$$P_s/T_s = ik f_0''/f_0''', \quad (3.2)$$

where  $\phi, f$  are the so-called 'inviscid' and 'viscous' solutions of the Orr-Sommerfeld equation. The subscript zero denotes evaluation at the surface of the liquid. The restriction  $\epsilon = k\delta = (\nu_a k^2/U_0')^{\frac{1}{2}} \ll 1$  is assumed to hold so that the

thickness of the friction layer,  $\delta$ , is small compared with the wavelength. Introducing the complex functions:

$$w = [1 + (U'_0/c)(\phi'_0/\phi'_0)]^{-1}, \tag{3.3}$$

$$\mathcal{F}(z_0) = [1 + (U'_0/c)(f'_0/f'_0)]^{-1}, \tag{3.4}$$

$$G(z_0) = f''_0/if'''_0, \tag{3.5}$$

with  $z_0 = c_0 k^{1/3}/U'^{2/3}_0 \nu_a^{1/3}$ , solutions for  $P_s, T_s$  may be obtained. The required part of  $P_s$  is

$$\begin{aligned} \mathcal{I}(P_s) = & \frac{cU_0}{|\mathcal{F}-w|^2} \{ (w_i - \mathcal{F}_i) + \epsilon z_0 [\mathcal{F}_i(\mathcal{F}_r - w_r) + (\mathcal{F}_r - 1)(w_i - \mathcal{F}_i)] \\ & + \epsilon z_0 (\frac{1}{2}R_w)^{1/2} \{ (\hat{w}_i - \mathcal{F}_i)[(\mathcal{F}_r - 1)(K_r + K_i) - \mathcal{F}_i(K_r - K_i)] \\ & + (\mathcal{F}_r - w_r)[(\mathcal{F}_r - 1)(K_i - K_r) + \mathcal{F}_i(K_r + K_i)] \}. \end{aligned} \tag{3.6}$$

The effect of the surface film is seen in the terms with  $\epsilon z_0$  as a factor; they arise from the inclusion of the horizontal velocity of the liquid in the boundary conditions. On the whole, the presence of the film has little effect on  $\mathcal{I}(P_s)$  ( $\epsilon$  is assumed small).

The tangential stress contribution to (2.7) is

$$(iT_s K) = \frac{\epsilon c U'_0}{|\mathcal{F}-w|^2} [(\mathcal{F}_r - w_r)(G_r K_r - G_i K_i) + (w_i - \mathcal{F}_i)(G_i K_r + G_r K_i)] \tag{3.7}$$

and even in the absence of film, this component is of second order. This is easily seen from (3.2), (3.5); since  $\epsilon \ll 1$  and  $G(z_0)$  is never more than about 2.3 (Miles 1962, figure 1),  $T_s$  is generally negligible compared with  $P_s$ . Through  $K_r, K_i$  the effect of  $\mathcal{I}(iT_s K)$  is reduced even further with a surface film.

The functions  $\mathcal{F}, w$  are dependent on the form of the velocity  $U \equiv U(y)$  in the gas phase. Miles (1962) used a linear-logarithmic profile

$$U(y) = (U_*^2/\nu_a)y \quad (0 < y \leq y_1), \tag{3.8}$$

$$= (U_*/\kappa) \log(U_*y/\nu_a) + K_1 U_*, \tag{3.9}$$

where  $\kappa, K_1$  are constants. The friction velocity is given by  $U_*^2 = U'_0 \nu_a = \tau_0/\rho_a$ , where  $\tau_0$  is the steady stress on the surface. A conservative value of the velocity at the edge of the laminar sublayer ( $y = y_1$ ) is  $U_1 = 5U_*$  and it is convenient to note that if  $U_\infty$  is the wind velocity far from the interface,  $U_\infty = U_*/c_f^{1/2}$  so that with a typical value of 0.0025 for  $c_f$  (Francis 1954; Fitzgerald 1963),  $U_\infty \doteq 20U_*$ .

A disadvantage with this profile is that at the edge of the laminar sublayer there is a discontinuity in  $U''$ . The function  $w_i$  is proportional to  $U''_c$  (where the subscript  $c$  denotes evaluation at the critical layer). Thus if the critical layer coincides with the edge of the laminar sublayer incorrect values of  $w_i$  will be calculated. (In fact Miles put  $w_i = 0$  here, which accounts for the anomalous results in this area of his curves (1962, figures 5a, 5b).) Accordingly we have used in this region the profile of Deissler (1955)

$$u^+ = \int_0^{s^+} \{1 + n^2 u^+ s^+ [1 - \exp(-n^2 u^+ s^+)]\} ds^+, \tag{3.10}$$

where  $u^+ = U/U_*, s^+ = yU_*/\nu_a$  and  $n$  is a constant, equal to 0.124 for a smooth surface. This gives continuous values of  $U''$ . The function  $w_r$  is relatively in-

sensitive to this change in velocity profile, and  $\mathcal{F}_i$ ,  $\mathcal{F}_r$  depend only on  $U'_0$ , to the first order of approximation.

#### 4. Results and discussion

The only system of practical interest at present is that of air-water. Figure 1 is a dimensional plot of the critical wind velocity as a function of wavelength with various values of the surface elasticity for this case. (The surface tension in the

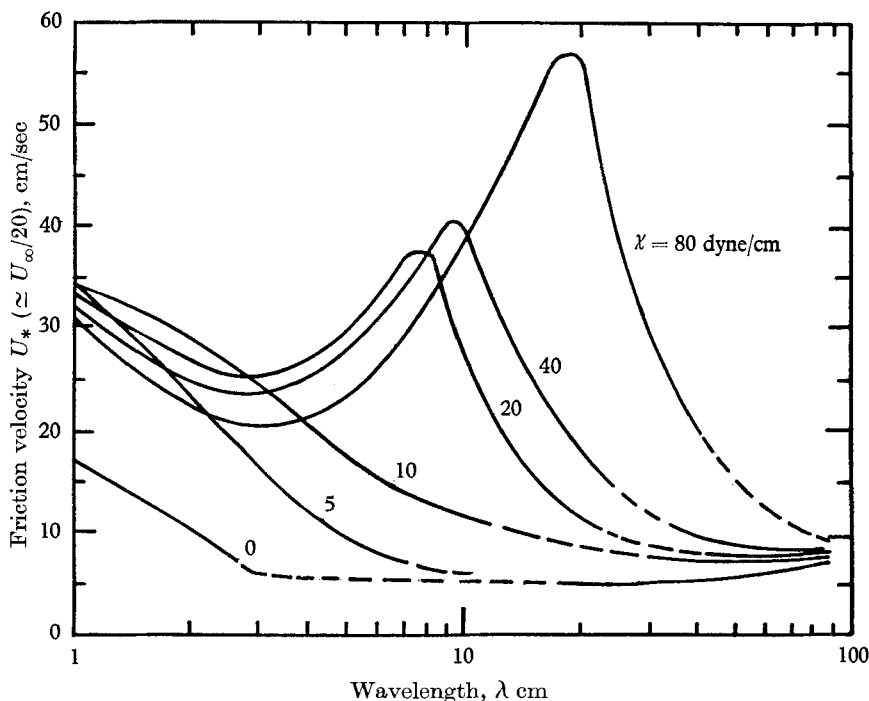


FIGURE 1. Dimensional neutral stability curves for air-water, at various values of the surface elasticity. The dashed lines indicate where the critical layer is within the transition region outside the viscous sublayer.

presence of a surface film is bound to be different to that of pure water, but this is only important at short wavelengths; we have used  $\sigma = 73$  dyne/cm throughout.) We have put an upper limit of 80 dyne/cm on  $\chi$  which may be slightly conservative (Davies & Rideal 1961). Condensed monolayers, which form islands or clumps of surfactant floating on the liquid surface, are excluded.

The effect of  $\chi$  is quite marked, and it is clear that even a film of very low elasticity will have a considerable effect at very short wavelengths. In the region of 1 to 5 cm, the curves are close together and it seems that a film for which  $\chi = 10$  dyne/cm say, is just as effective as an inextensible film,  $\chi = \infty$ . This is a result of the maximum in the liquid damping which occurs at  $\mathcal{D}(R_w/2)^{\frac{1}{2}} = 1$ , or for gravity waves  $\chi^4 k^5 / 4\rho^4 g^3 \nu^2 = 1$ . Thus the value of the most effective  $\chi$  varies almost directly as the wavelength  $\lambda$ . This point is better illustrated in figure 2, which gives the wave speed  $c_0$ , the critical free stream wind velocity  $U_\infty$  for a clean surface, and the value  $\chi_{\max}$  which produces maximum damping, as a func-

tion of the wavelength. The damping in the clean liquid varies as  $[R_w^{-1}kc_0]$ , which is reflected in the high critical wind speed at small  $\lambda$ ; for large  $\lambda$ ,  $c_0$  increases and for a critical layer mechanism to apply, the wind velocity must always exceed  $c_0$  while approaching it asymptotically.

For a typical surfactant, the above theory can explain why, to damp waves of a particular wavelength, there is an optimum concentration (Aitken 1882; Francis 1967). In general as  $\chi$  increases, so does the critical wind speed for fixed  $\lambda$ . However,  $\chi$  does not necessarily increase with surface concentration, and although the surface tension generally decreases monotonically with increasing  $\Gamma_0$  (Davies & Rideal 1961, chapter 5),  $\chi = -\partial\sigma/\partial\ln\Gamma_0$  is typically small for low  $\Gamma_0$ , building up to a maximum and then declining to zero at large  $\Gamma_0$  (Lucassen & Hansen 1966). There is, as Aitken says, just one quantity which will give the maximum effect; more or less will not do so well.

Although we have no precise knowledge as to the values of  $\chi$  for the various animal, vegetable and mineral oils used by Franklin (1774) and Aitkin (1882), it is very probable that they would form monolayers if allowed to spread on water, for which  $\chi = 10$  dyne/cm would be a conservative *minimum* value (Davies & Rideal 1961, p. 265). Figure 1 shows that even with such a small value as this, large increases would be obtained in the critical wind speed at low  $\lambda$ , so that it should be possible to dampen waves of length 1 to 3 cm very easily by a small amount of a surface impurity. Incidentally, Aitken gives a table of oils claimed to be ranked in order of their ability to calm waves; in fact they are arranged in order of increasing surface tension when spread on water. Now for an ideal adsorbed monolayer,  $\chi$  is equal to the surface pressure, which is the decrease of surface tension of the liquid due to the presence of the surface film. (For oils the surface pressure ranges from about 15 dyne/cm *upwards*.) So a table of oils of increasing surface tension will probably also be one of decreasing  $\chi$ . Fortunately, Aitken's table is probably correct.

Our results may be compared to some extent with the experiments of Fitzgerald (1963). He measured the surface stress  $\tau_0$  for flow of air over water in a rectangular channel, with and without surface films. He found that for mean velocities above about 500 cm/sec, waves grew even in the presence of a cetyl alcohol film, although for lower velocities, waves were suppressed. At 500 cm/sec, his measured value of  $\tau_0$  was 0.7 dyne/cm<sup>2</sup> approximately which gives  $U_* \doteq 25$  cm/sec. This is in reasonable agreement with our results which show that for a surface film of moderate elasticity ( $\chi = 20$  dyne/cm say) the minimum critical wind speed is  $U_* = 25$  cm/sec for a wavelength of 2–3 cm. At this wind speed, we would also expect waves of length greater than about 10 cm. However, Fitzgerald inferred the value 500 cm/sec from the change in slope of a graph of  $\tau_0$  vs.  $U_*^2$ , and it is likely that the shear stress is much more susceptible to the presence of shorter waves than to the longer ones. A difficulty of interpretation also arises with the results of Keulegan (1951) who found that in the presence of soap, he could suppress waves in a wind tunnel at wind speeds up to 1200 cm/sec or  $U_* \doteq 60$  cm/sec. On our results this seems rather unlikely, and of course this is greatly in excess of Fitzgerald's figure. We would certainly expect a surface film to damp capillary ripples but above  $U_* = 6$  cm/sec, gravity waves would always

be expected to develop. But the water depth in Keulegan's case was not very great (4–14.5 cm) so perhaps long waves were damped by the lower-boundary effect. We also note the possibility that in this type of experiment (but not in Keulegan's) the long waves may be longer than the apparatus. Also the rate of growth of short waves is high compared with that for the long ones.

## 5. Conclusion

This study has shown that, with a surface-active monolayer of experimentally attainable characteristics, the critical wind speed for wave generation may be

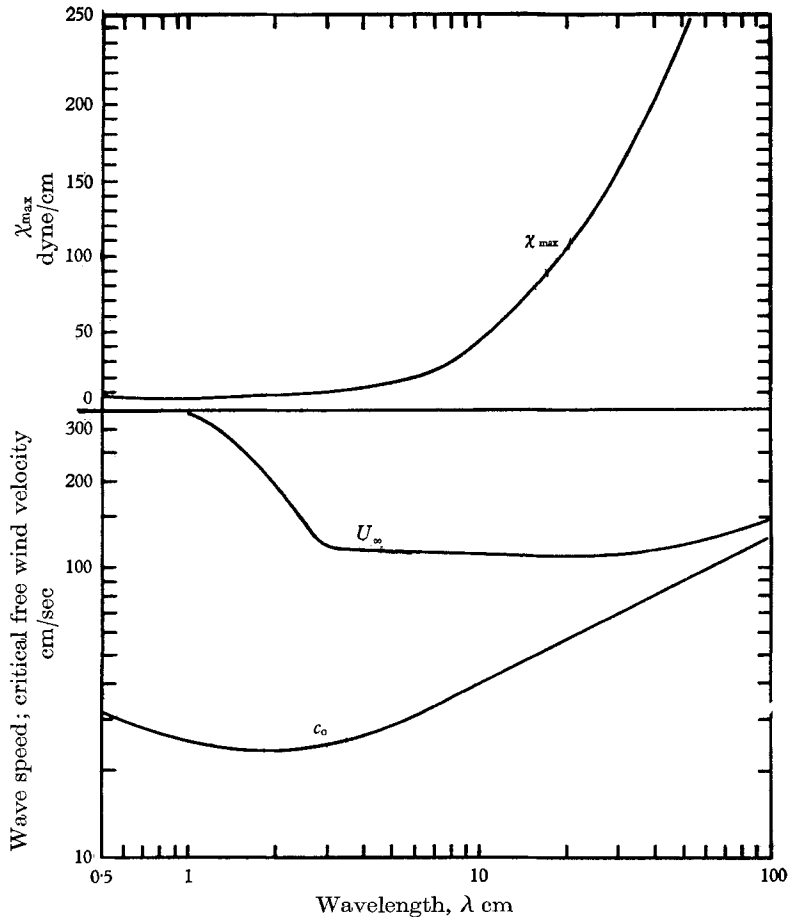


FIGURE 2. Above: values of the surface compressibility  $\chi_{\max}$  for which the damping in the liquid is a maximum, at a given wavelength. Below: the critical free wind speed ( $U_{\infty} \approx 20U_*$ ) and the wave velocity  $c_0$  for  $\chi = 0$ .

increased by a factor of up to ten depending on  $\chi$  and  $k$ , and that for the shortest wavelengths, a very small value of surface elasticity will increase the critical wind speed by a factor of two. For waves greater than about 1 m in length, no known surface film would have any effect. With this theory we have been able to explain almost all the available experimental observations, but admittedly these are mainly qualitative.



As far as the liquid and the surface film are concerned, the theory is on relatively firm ground, but in the gas phase the theory is more open to doubt. However, recent work indicates that the shear flow mechanism is probably valid, at least for the capillary waves which are most important in our problem (Miles 1967*b*; Shemdin & Hsu 1967). But in any case providing wave generation occurs by a critical layer mechanism, errors in the details will not invalidate the general results we have obtained. As figure 2 shows, damping of capillary waves will always be most effected by films of low  $\chi$ , and there are no known surface films which could suppress longer waves.

We would like to thank Dr C. H. Ellen, Prof. J. R. D. Francis and Dr J. A. Kitchener of Imperial College, and Dr W. W. Mansfield of C.S.I.R.O. Melbourne for fruitful discussions. One of us (J. C. G.) is grateful to the Consejo Nacional de Investigaciones Cientificas y Tecnicas, Argentina, for a Research Scholarship.

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*Note added in proof*

(1) Our calculations show that the wavelength of the first waves to appear as the wind velocity is increased over a clean water surface should be about 20 cm (figure 5). This is at variance with the value of about 4 cm obtained from Miles (1962), and is due to the use of the Deissler velocity profile (3.10) in place of the linear-logarithmic distribution used there. But for  $\lambda$  between 3 and 20 cm, the critical wind speed is almost unchanged, and since the growth rate  $kc_i$  increases with decreasing wavelength, the shorter waves would probably be seen in experiments.

(2) In a paper soon to be published in *Chemical Engineering Science*, Prof. J. T. Davies of the University of Birmingham has reported measurements of the amplitudes of wind-generated capillary waves. The observed growth rates are in fair agreement with equation (2.7).